LAGRANGE COLLEGE

Citations Journal of Undergraduate Research
2020, Volume 17

Using Boltzmann Transport Theory to Compute Field Interaction Between Dark Matter and Dark Energy

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Science, Chemistry and Physics

Abstract

Within the literature, many different parameterizations of couplings between dark energy (DE) and dark matter (DM) as fluids in the continuity equation have been studied and examined, and observational data can constrain these parameterizations. Instead of a fluid coupling, we present here a study of DE-DM coupling as fields, which is a more fundamental description of interaction, and our method is novel in that we utilize the Boltzmann Transport equation to calculate the interaction. Since the equation-of-state parameter for DE is observationally negative, it is necessary to use a distribution function for DE in the Boltzmann Transport equation that leads to a negative equation-of-state parameter, which neither Bose-Einstein nor Fermi-Dirac distributions can supply. We utilize an effective distribution function derived from quantum field theory in curved spacetime that accounts for our negative state parameter. We present and examine our results for a Yukawa-type coupling between DE and DM, and we show how the DE-DM interaction term in the continuity equation depends on different parameters such as the DE and DM mass.

Introduction

Our universe is expanding, and since several billion years after our universe began, it has been expanding outward more quickly with the progression of time; our universe is not only expanding but also accelerating, according to observations. We call the cause of this acceleration "dark energy" (DE). It is called "dark" because it is not observed via the electromagnetic spectrum. Instead, DE is detected via the "anti-gravitational" effect that it has on normal matter, causing the universe to accelerate outward rather than to gravitationally collapse inward upon itself. DE accounts for ~68% of the contents of our universe.

When observing our universe and how celestial bodies interact within it, cosmologists saw a discrepancy between their calculations and observations. In order for observations to make sense, there must be more mass present than is actually electromagnetically detected. This matter was called "dark matter" (DM) for its apparent lack of luminosity and lack of reaction to electromagnetic waves. DM still has the same gravitational effects as those of normal matter. DM makes up ~27% of our universe.

Our universe is continuously expanding and accelerating, and it is unknown if there is any interaction between dark matter (DM) and dark energy (DE). The standard way of modeling the contents of our universe is to treat them as perfect fluids, and conservation of energy and momentum in an expanding universe implies

$$\nabla_{\alpha} T^{\mu\nu} = 0 \quad \rightarrow \quad \sum_i \left(\dot{\rho}_i + 3H(\rho_i + p_i)\right) = 0\,, \tag{1}$$

where ρ is the fluid energy density, p is the fluid pressure, and H is the Hubble parameter, which describes the expansion rate of our universe. We also use natural units here and throughout, so that $c = \hbar = k_B = 1$. In late cosmological times, DE and DM dominate over other constituents of our universe, such as radiation and ordinary matter. Therefore, it follows from Eq. (1) that

$$\dot{\rho}_{DM} + 3H(\rho_{DM} + p_{DM}) = Q, \quad \dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = -Q\,. \tag{2}$$

Calculating the Interaction

Typically, works in the cosmology literature assume that $Q = 0$, which means no interaction between DM and DE. When an interaction is assumed, an *ad hoc* parameterization is applied that treats DM and DE as fluids rather than fields. Instead, we model interaction on a more fundamental level in that we treat DE and DM as scalar fields. Since we are assessing the interaction as fields, we assume a standard Yukawa-type field interaction,

$$\frac{1}{2} g\psi^2\phi \tag{3}$$

where g is our coupling constant, ψ is the DM field, and ϕ is the DE field. Assuming a 2-to-2 conversion, a pair of two dark matter particles and two dark energy particles can convert one to the other [1, 2]. It can then be shown using the Boltzmann Transport equation that [3]

$$Q = MB, \tag{4}$$

where M is the DM mass and B is given by

$$B = \int \frac{d^3k_1}{(2\pi)^3 2\sqrt{k_1^2+M^2}} \frac{d^3k_2}{(2\pi)^3 2\sqrt{k_2^2+M^2}} \frac{d^3k_3}{(2\pi)^3 2\sqrt{k_3^2+m^2}} \frac{d^3k_4}{(2\pi)^3 2\sqrt{k_4^2+m^2}} \times (2\pi)^4\delta^4(k_1+k_2-k_3-k_4)\left[f_\psi(k_1)f_\psi(k_2)-f_\phi(k_3)f_\phi(k_4)\right]|\mathscr{M}|^2. \tag{5}$$

Within Eq. (5), we define m as the DE mass and M as the DM mass. Furthermore, our DM distribution function, f_ψ, is the Bose-Einstein distribution:

$$f_\psi(k) = \frac{1}{e^{\sqrt{k^2+M^2}/T}-1}. \tag{6}$$

The DE distribution function, f_ϕ, is obtained in [3]. Due to its length, we do not reproduce it here, but it is a function that depends on the DE mass m, the scale factor of the expanding universe a, the DE non-minimal coupling parameter ξ, and momentum. The parameter a is a function of time that describes the expansion rate of the universe dominated by DE and DM, and ξ is the coupling between the curvature of spacetime and DE. $\mathscr{M}$ refers to the DE-DM scattering amplitude, which is given by

$$|\mathscr{M}|^2 = g^4\left(\frac{1}{M^2-t}+\frac{1}{M^2-u}\right)^2. \tag{7}$$

In the center-of-mass frame, t and u are defined as

$$t = -2\sqrt{k_3^2+m^2}\sqrt{k_1^2+M^2}+2k_1k_3\cos\theta+m^2+M^2 \tag{8}$$

$$u = 2m^2+2M^2-4M^2\left(\left(\frac{k_1}{M}\right)^2+1\right)-t. \tag{9}$$

We calculate Q and plot the result in Fig. 1, and we plot Q as a function of the scale factor a. We use $a(t)$ for a universe dominated by DE, which is valid for late cosmological times:

$$a(t) = 1+(1+w)\left(6\pi G\rho_{DE0}\right)^{1/2}(t-t_0)^{2(1+w)/3}, \tag{10}$$

where ρ_{DE0} is the observational value of the present-day DE energy density, $w = -0.9$, and t_0 is the present-day time, all of which are consistent with observational data [3].

Results

For our plot in Fig. 1, the range of our horizontal axis corresponds to the time period over which dark energy dominates the universe, and $a = 1.0$ corresponds to present time. Our vertical axis shows the coupling Q between DM and DE in units of eV^5. Furthermore, notice that the function

$$M = 10^9\text{eV},\ m = 1.567\times10^{-54}\text{eV},\ \xi = 0.176,\ g = 0.1\text{eV}. \tag{11}$$

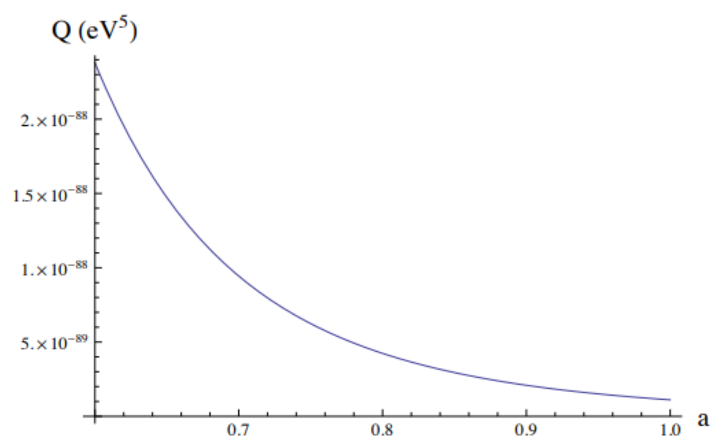

Figure 1: Q versus a.

continually decreases in value as it approaches present time and that Q has a low interaction value. For this plot, we have set our parameters to ones in agreement with observational constraints:

In our analysis, we let g range from 0 eV to 1 eV. Also, m is constrained from observable data in [3], and we use a reasonable range for M, from 10^9 eV to 10^{12} eV. For these parameter ranges, the order of magnitude of Q can be anywhere from 10^{-100} eV^5 to 10^{-80} eV^5. For comparison, a typical value for a parameterization of Q that treats DM and DE as fluids is $|Q| \sim 10^{-45}$ eV^5 [3].

Conclusion

We have demonstrated a novel method of calculating the interaction in the continuity equation, Q, between DE and DM involving the Boltzmann Transport equation. Treating DE and DM as fields is more fundamental compared to their typical treatment in the cosmological literature as fluids, and we applied our method to a standard Yukawa-type coupling with 2-to-2 conversion between DE and DM. We found that the interaction strength Q from the continuity equation is relatively small for this coupling. The results show the viability of this method of calculating Q, and it can be applied to other kinds of field couplings as well.

[1] Jeremy Bernstein, *Kinetic theory in the expanding universe*, Cambridge University Press (1988).
[2] Edward W. Kolb and Michael S. Turner, *The Early Universe*, Westview Press (1990).
[3] Kevin J. Ludwick, arXiv: 1909.10890 [gr-qc].